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On a Method of Solving Grid Structure by Means of Fourier Transforms Concerning Finite Integration

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Abstract

This paper presents inversion formulas regarding a kind of finite Fourier Transforms which are derived from finite integration, together with the related formulas which plays some part in finding finite Fourier transformations out of finite difference equations. Making use of them, the stress problem of the grid structure, working like plate, which is expressed by three simultaneous finite difference equations, is solved in a similar way as the method of finite Fourier transforms goes for solving the boundary problems of continuous media.

1. Preliminary Formulas

Since the differential equation has a basic similarity with the finite difference equation, it is expectable that the principle of an analitical method which is good for the one should analogically be valid for the other. From this point of view, let us look at how the method of finite sine and cosine transforms[1] plays its part in solving the differential equation, and try to develope a similar method for the finite difference equation.

(a) Inversion Formulas with Respect to Finite Sine and Cosine Series

Denoting the symbolic notations as

$$S_i[f(x)] = \sum_{x=1}^{m-1} f(x) \sin \frac{i\pi x}{m} \tag{1}$$

$$C_i[f(x)] = \sum_{x=1}^{m-1} f(x) \cos \frac{i\pi x}{m}, \tag{2}$$

we have the inversion formulas[2),3)] coupled with the above, as follows:

$$f(x) = \frac{2}{m} \sum_{i=1}^{m-1} S_i[f(x)] \sin \frac{i\pi x}{m} \qquad (0 \angle x \angle m) \tag{3}$$

$$f(x) = \sum_{i=0}^{m} R_i \cos \frac{i\pi x}{m}, \qquad (0 \leqq x \leqq m), \tag{4}$$

where

$$R_0 = \frac{1}{m} \left\{ C_0[f(x)] + \frac{1}{2} f(m) + \frac{1}{2} f(0) \right\},$$

$$R_i = \frac{1}{m} \{2C_i[f(x)] + f(m) \times (-1)^i + f(0)\},$$

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$$R_m = \frac{1}{m}\left\{C_m[f(x)] + \frac{1}{2}f(m)(-1)^m + \frac{1}{2}f(0)\right\},$$

$$x, i = 0, 1, 2, 3, \cdots m.$$

(b) **Related Formulas**

$$S_i[\Delta^2 f(x-1)] = -\sin\frac{i\pi}{m}\left\{(-1)^i f(m) - f(0)\right\} - D_i S_i[f(x)], \tag{5}$$

$$C_i[\Delta^2 f(x-1)] = \Delta f(m-1)(-1)^i - \Delta f(0) - D_i\left\{\frac{1}{2}f(m)(-1)^i + \frac{1}{2}f(0) + C_i[f(x)]\right\}, \tag{6}$$

$$S_i[f(x+1) - f(x-1)] = -2\sin\frac{i\pi}{m}\left\{\frac{1}{2}f(m)(-1)^i + \frac{1}{2}f(0) + C_i[f(x)]\right\}, \tag{7}$$

$$C_i[f(x+1) - f(x-1)] = -\{\Delta f(m-1)(-1)^i + \Delta f(0)\} + \left(1 + \cos\frac{i\pi}{m}\right)\{f(m)(-1)^i - f(0)\} + 2\sin\frac{i\pi}{m}S_i[f(x)], \tag{8}$$

where

$$\Delta f(x) = f(x+1) - f(x), \quad D_i = 2\left(1 - \cos\frac{i\pi}{m}\right).$$

(c) **Inversion Formulas for Function of Two Variables**

Let δ_{xy} and θ_{xy} be the functions of the two variables x and y, then the inversion formulas (1) and (3), and (2) and (4) can be extended to the following forms:

$$S_i S_r[\delta_{x\cdot y}] = \sum_{x=1}^{m-1}\sum_{y=1}^{n-1}\delta_{x\cdot y}\sin\frac{i\pi x}{m}\sin\frac{i\pi y}{n}, \tag{9}$$

$$\delta_{x\cdot y} = \frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}S_i S_r[\delta_{x\cdot y}]\sin\frac{i\pi x}{m}\sin\frac{i\pi y}{n}, \tag{10}$$

and

$$C_i S_r[\theta_{x\cdot y}] = \sum_{x=1}^{m-1}\sum_{y=1}^{n-1}\theta_{x\cdot y}\cos\frac{i\pi x}{m}\sin\frac{i\pi y}{n}, \tag{11}$$

$$\theta_{x\cdot y} = \sum_{i=0}^{m}\sum_{r=1}^{n-1}\Theta_{ir}\cos\frac{i\pi x}{m}\sin\frac{i\pi y}{n}, \tag{12}$$

where

$$\Theta_{0r} = \frac{2}{mn}\left\{C_0 S_r[\theta_{x\cdot y}] + \frac{1}{2}S_r[\theta_{m\cdot y}] + \frac{1}{2}S_r[\theta_{0\cdot y}]\right\}$$

$$\Theta_{ir} = \frac{4}{mn}\left\{C_i S_r[\theta_{x\cdot y}] + \frac{1}{2}S_r[\theta_{m\cdot y}](-1)^i + \frac{1}{2}S_r[\theta_{0\cdot y}]\right\}$$

$$\Theta_{m\cdot r} = \frac{2}{mn}\left\{C_m S_r[\theta_{x\cdot y}] + \frac{1}{2}S_r[\theta_{m\cdot y}](-1)^m + \frac{1}{2}S_r[\theta_{0\cdot y}]\right\},$$

$0 < x < m, \quad 0 < y < n$ for (9), (10);

$0 \leqq x \leqq m, \quad 0 < y < n$ for (11), (12).

2. Finite Difference Equations of Grid Plate

The grid work system shown in Fig. 1 which consists of two systems of parallel beams spaced equal distances λ_1 and λ_2 apart in x and y directions respectively, and rigidly connected at their point of intersection, is here to be called "grid plate".

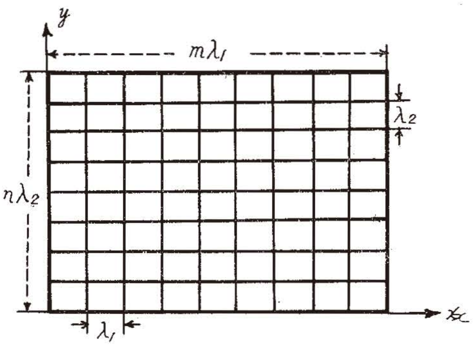

Fig. 1. Plan of grid plate

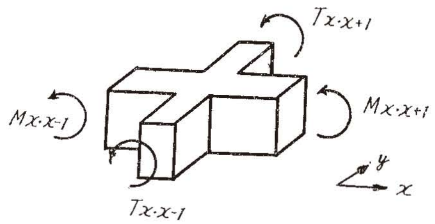

Fig. 2. Moments about y axis at x, y point

Denote the flexural and torsional rigidities of the componet beams by

$\lambda_1 K_{10}$; the flexural rigidity of the edge beams in x direction,
$\lambda_1 K_1$: the flexural rigidity of any other beams in x direction,
$\lambda_1 B_{10}$: the torsional rigidity of the edge beams in x direction,
$\lambda_1 B_1$: the torsional rigidity of any other beams in x direction,
$\lambda_2 K_{20}$: the flexural rigidity of the edge beams in y direction,
$\lambda_2 K_2$: the flexural rigidity of any other beams in y direction,
$\lambda_2 B_{20}$: the torsional rigidity of the edge beams in y direction,
$\lambda_2 B_2$: the torsional rigidity of any other beams in y direction,

and let $\theta_{x\cdot y}$ be a rotation about y axis, $\theta'_{x\cdot y}$ be a rotaion about x axis, and $\delta_{x\cdot y}$ be a deflection in the downward direction at x, y point, then the slope-deflection equation yields the bending moments of the beam in x direction at x, y point, as follows:

$$M_{x\cdot x+1} = 2K_1 \{2\theta_{x\cdot y} + \theta_{x+1\cdot y} - 3(\delta_{x+1\cdot y} - \delta_{x\cdot y})/\lambda_1\}, \tag{13}$$

$$M_{x\cdot x-1} = 2K_1 \{2\theta_{x\cdot y} + \theta_{x-1\cdot y} - 3(\delta_{x\cdot y} - \delta_{x-1\cdot y})/\lambda_1\}, \tag{14}$$

in which the first subscription represents the point of intersection and the second one shows the component beam where the bending moment occurs.

The torsional moments of the beam in y direction at the same intersection, take the following form:

$$T_{y\cdot y+1} = B_2(\theta_{x\cdot y} - \theta_{x\cdot y+1}), \tag{15}$$

$$T_{y\cdot y-1} = B_2(\theta_{x\cdot y} - \theta_{x\cdot y-1}), \tag{16}$$

in which the first equation stands for the torsional moment of the x-th beam in y direction between y and $y+1$, and the second one for the torsional moment of the same beam between $y-1$ and y. By adding up the equations (13), (14), (15) and (16), equilibrium of moments about y axis at the intersection x, y can be expressed by

$$2K_1\left\{\Delta_x^2\theta_{x-1\cdot y}+6\theta_{x\cdot y}-\frac{3}{\lambda_1}(\delta_{x+1\cdot y}-\delta_{x-1\cdot y})\right\}-B_2\Delta_y^2\theta_{x\cdot y-1}=0\,, \tag{17}$$

where

$$\Delta_x^2\theta_{x-1\cdot y}=\theta_{x+1\cdot y}-2\theta_{x\cdot y}+\theta_{x-1\cdot y}\,.$$

In a similar way, making use of the bending moments of the x-th beam in y direction

$$M'_{y\cdot y+1}=2K_2\{2\theta'_{x\cdot y}+\theta'_{x\cdot y+1}-3(\delta_{x\cdot y+1}-\delta_{x\cdot y})/\lambda_2\}\,, \tag{18}$$

$$M'_{y\cdot y-1}=2K_2\{2\theta'_{x\cdot y}+\theta'_{x\cdot y-1}-3(\delta_{x\cdot y}-\delta_{x\cdot y-1})/\lambda_2\}\,, \tag{19}$$

and the torsional moments of the y-th beam in x direction

$$T'_{x\cdot x+1}=B_1(\theta'_{x\cdot y}-\theta'_{x+1\cdot y})\,, \tag{20}$$

$$T'_{x\cdot x-1}=B_1(\theta'_{x\cdot y}-\theta'_{x-1\cdot y})\,, \tag{21}$$

we can write equilibrium of moments about y axis at the intersection x, y as follows:

$$2K_2\left\{\Delta_y^2\theta'_{x\cdot y-1}+6\theta'_{x\cdot y}-\frac{3}{\lambda_2}(\delta_{x\cdot y+1}-\delta_{x\cdot y-1})\right\}-B_1\Delta_x^2\theta'_{x-1\cdot y}=0\,. \tag{22}$$

The remainning condition at the point x, y is equilibrium of vertical forces: that is

$$(M_{x+1\cdot x}+M_{x\cdot x+1}-M_{x\cdot x-1}-M_{x-1\cdot x})/\lambda_1$$
$$+(M'_{y+1\cdot y}+M'_{y\cdot y+1}-M'_{y\cdot y-1}-M'_{y-1\cdot y})/\lambda_2+q_{x\cdot y}=0\,,$$

from which we have

$$\frac{6K_1}{\lambda_1}\left\{\theta_{x+1\cdot y}-\theta_{x-1\cdot y}-\frac{2}{\lambda_1}\Delta_x^2\delta_{x-1\cdot y}\right\}$$
$$+\frac{6K_2}{\lambda_2}\left\{\theta'_{x\cdot y+1}-\theta'_{x\cdot y-1}-\frac{2}{\lambda_2}\Delta_y^2\delta_{x\cdot y-1}\right\}+q_{x\cdot y}=0\,, \tag{23}$$

where $q_{x\cdot y}$ is a concentrated load acting on the intersection x, y. In these discussion the load is assumed to act concentratedly only on the point of intersection.

3. Boundary Conditions

The equilibrium conditions for the four edge beams, should take different forms from what we have in the equations (17), (22) and (23), because the edge intersection consists of the three component beams while the four component beams meet at the inside intersections. For these edge beams, the equilibriums of moments about x and y axes, and the equilibrium of vertical forces are expressed by

$$2K_2\left\{2\theta'_{x\cdot 0}+\theta'_{x\cdot 1}-\frac{3}{\lambda_2}(\delta_{x\cdot 1}-\delta_{x\cdot 0})\right\}-B_{10}\Delta_x^2\theta'_{x-1\cdot 0}+\mathfrak{M}'_0=0\,, \tag{24}$$

$$2K_{10}\left\{\Delta_x^2\theta_{x-1.0}+6\theta_{x\cdot 0}-\frac{3}{\lambda_1}(\delta_{x+1\cdot 0}-\delta_{x-1\cdot 0})\right\}-B_2(\theta_{x\cdot 1}-\theta_{x\cdot 0}+\mathfrak{T}_0=0\,, \tag{25}$$

$$\frac{6K_{10}}{\lambda_1}\left\{\theta_{x+1\cdot 0}-\theta_{x-1\cdot 0}-\frac{2}{\lambda_1}\Delta_x^2\delta_{x-1\cdot 0}\right\}$$
$$+\frac{6K_2}{\lambda_2}\left\{\theta'_{x\cdot 1}+\theta'_{x\cdot 0}-\frac{2}{\lambda_2}(\delta_{x\cdot 1}-\delta_{x\cdot 0})\right\}+q_{x\cdot 0}=0\,, \tag{26}$$

for $y=0$;

$$2K_2\left\{2\theta'_{x\cdot n}+\theta'_{x\cdot n-1}-\frac{3}{\lambda_2}(\delta_{x\cdot n}-\delta_{x\cdot n-1})\right\}-B_{10}\Delta_x^2\theta'_{x-1\cdot n}+\mathfrak{M}'_{2n}=0\,, \tag{27}$$

$$2K_{10}\left\{\Delta_x^2\theta_{x-1\cdot n}+6\theta_{x\cdot n}-\frac{3}{\lambda_1}(\delta_{x+1\cdot n}-\delta_{x-1\cdot n})\right\}+B_2(\theta_{x\cdot n}-\theta_{x\cdot n-1})+\mathfrak{T}_n=0\,, \tag{28}$$

$$\frac{6K_{10}}{\lambda_1}\left\{\theta_{x+1\cdot n}-\theta_{x-1\cdot n}-\frac{2}{\lambda_1}\Delta_x^2\delta_{x-1\cdot n}\right\}$$
$$-\frac{6K_2}{\lambda_2}\left\{\theta'_{x\cdot n}+\theta'_{x\cdot n-1}-\frac{2}{\lambda_2}(\delta_{x\cdot n}-\delta_{x\cdot n-1})\right\}+q_{x\cdot n}=0\,, \tag{29}$$

for $y=n$;

$$2K_1\left\{2\theta_{0\cdot y}+\theta_{1\cdot y}-\frac{3}{\lambda_1}(\delta_{1\cdot y}-\delta_{0\cdot y})\right\}-B_{20}\Delta_y^2\theta_{0\cdot y-1}+\mathfrak{M}_0=0\,, \tag{30}$$

$$2K_{20}\left\{\Delta_y^2\theta'_{0\cdot y-1}+6\theta'_{0\cdot y}-\frac{3}{\lambda_2}(\delta_{0\cdot y+1}-\delta_{0\cdot y-1})\right\}-B_1(\theta'_{1\cdot y}-\theta'_{0\cdot y})+\mathfrak{T}'_0=0\,, \tag{31}$$

$$\frac{6K_{20}}{\lambda_2}\left\{\theta'_{0\cdot y+1}-\theta'_{0\cdot y-1}-\frac{2}{\lambda_2}\Delta_y^2\delta_{0\cdot y-1}\right\}$$
$$+\frac{6K_1}{\lambda_1}\left\{\theta_{0\cdot y}+\theta_{1\cdot y}-\frac{2}{\lambda_1}(\delta_{1\cdot y}-\delta_{0\cdot y})\right\}+q_{0\cdot y}=0\,, \tag{32}$$

for $x=0$;

$$2K_1\left\{2\theta_{m\cdot y}+\theta_{m-1\cdot y}-\frac{3}{\lambda_1}(\delta_{m\cdot y}-\delta_{m-1\cdot y})\right\}-B_{20}\Delta_y^2\theta_{m\cdot y-1}+\mathfrak{M}_m=0\,, \tag{33}$$

$$2K_{20}\left\{\Delta_y^2\theta'_{m\cdot y-1}+6\theta'_{m\cdot y}-\frac{3}{\lambda_2}(\delta_{m\cdot y+1}-\delta_{m\cdot y-1})\right\}+B_1(\theta'_{m\cdot y}-\theta'_{m-1\cdot y})+\mathfrak{T}'_m=0\,, \tag{34}$$

$$\frac{6K_{20}}{\lambda_2}\left\{\theta'_{m\cdot y+1}-\theta'_{m\cdot y-1}-\frac{2}{\lambda_2}\Delta_y^2\delta_{m\cdot y-1}\right\}$$
$$-\frac{6K_1}{\lambda_1}\left\{\theta_{m\cdot y}+\theta_{m-1\cdot y}-\frac{2}{\lambda_1}(\delta_{m\cdot y}-\delta_{m-1\cdot y})\right\}+q_{m\cdot y}=0\,, \tag{35}$$

for $x=m$;

where $\mathfrak{M}_0$ and $\mathfrak{M}_m$: external bending moments about y axis which act at any intersection of $x=0$ and $x=m$ respectively,

$\mathfrak{M}'_0$ and $\mathfrak{M}'_n$: external bending moments about x axis which act at any intersection of $y=0$ and $y=n$ respectively,

$\mathfrak{T}'_0$ and $\mathfrak{T}'_m$: external torques about x axis which act at any intersection of $x=0$ and $x=m$ respectively,

$\mathfrak{T}_0$ and $\mathfrak{T}_n$: external torques about y axis which act at any intersection of $y=0$ and $y=n$ respectively.

Using the above expressions, we can write the typical boundary conditions for the edge beam $x=0$, as follows

(a) simply supported edge

$$\mathfrak{M}_0=0\,, \qquad \mathfrak{T}_0=0\,, \qquad \delta_{0\cdot y}=0\,, \tag{36}$$

(b) hinged edge

$$\mathfrak{M}_0=0\,, \qquad \theta'_{0\cdot y}=0\,, \qquad \delta_{0\cdot y}=0\,, \tag{37}$$

(c) fixed edge

$$\theta_{0\cdot y}=0\,, \qquad \theta'_{0\cdot y}=0\,, \qquad \delta_{0\cdot y}=0\,, \tag{38}$$

(d) free edge

$$\mathfrak{M}_0=0\,, \qquad \mathfrak{T}_0=0\,, \qquad q_{0\cdot y}=0\,, \tag{39}$$

which shows that the three equations are needed to settle a physical state of the edge beam. Besides those equations from (24) to (35), the corner intersections lead to the results :

$$2K_{20}\left\{2\theta'_{0\cdot 0}+\theta'_{0\cdot 1}-\frac{3}{\lambda_2}(\delta_{0.1}-\delta_{0.0})\right\}+B_{10}(\theta'_{0\cdot 0}-\theta'_{1.0})+\mathfrak{M}'_{0\cdot 0}=0\,, \tag{40}$$

$$2K_{10}\left\{2\theta_{0\cdot 0}+\theta_{1\cdot 0}-\frac{3}{\lambda_1}(\delta_{1\cdot 0}-\delta_{0\cdot 0})\right\}+B_{20}(\theta_{0\cdot 0}-\theta_{0.1})+\mathfrak{M}_{0\cdot 0}=0\,, \tag{41}$$

$$\frac{6K_{10}}{\lambda_1}\left\{\theta_{1.0}+\theta_{0\cdot 0}-\frac{2}{\lambda_1}(\delta_{1.0}-\delta_{0\cdot 0})\right\}+\frac{6K_{20}}{\lambda_1}\left\{\theta'_{0\cdot 1}+\theta'_{0\cdot 0}-\frac{2}{\lambda^2}(\delta_{0\cdot 1}-\delta_{0\cdot 0})\right\}+q_{0\cdot 0}=0\,. \tag{42}$$

for $x=0,\ y=0$;

$$2K_{20}\left\{2\theta'_{0\cdot n}+\theta'_{0\cdot n-1}-\frac{3}{\lambda_2}(\delta_{0\cdot n}-\delta_{0\cdot n-1})\right\}+B_{10}(\theta'_{0\cdot n}-\theta'_{1\cdot n})+\mathfrak{M}'_{0\cdot n}=0\,, \tag{43}$$

$$2K_{10}\left\{2\theta_{0\cdot n}+\theta_{1\cdot n}-\frac{3}{\lambda_1}(\delta_{1\cdot n}-\delta_{0\cdot n})\right\}+B_{20}(\theta_{0\cdot n}-\theta_{0\cdot n-1})+\mathfrak{M}_{0\cdot n}=0\,, \tag{44}$$

$$\frac{6K_{10}}{\lambda_1}\left\{\theta_{1\cdot n}+\theta_{0\cdot n}-\frac{2}{\lambda_1}(\delta_{1\cdot n}-\delta_{0\cdot n})\right\}$$
$$-\frac{6K_{20}}{\lambda_2}\left\{\theta_{0\cdot n}+\theta_{0\cdot n-1}-\frac{2}{\lambda_2}(\delta_{0\cdot n}-\delta_{0\cdot n-1})\right\}+q_{0\cdot n}=0\,, \tag{45}$$

for $x=0$, $y=n$; and the equations at the corner points $(m\cdot 0)$, $(m\cdot n)$ could easily be written in a similar way.

4. Finite Fourier Transformations of $\theta_{x\cdot y}$, $\theta'_{x\cdot y}$, $\delta_{x\cdot y}$

Applying the symbolic operators C_iS_r, S_iC_r, and S_iS_r which are defined by (9) and (11), to the finite difference equations (17), (22) and (23) respectively, we have

$$S_r[\mathfrak{M}_m]\times(-1)^i+S_r[\mathfrak{M}_0]-\frac{3K_1}{\lambda_1}(4-D_i)\{S_r[\delta_{m\cdot y}](-1)^i-S_r[\delta_{0\cdot y}]\}$$
$$+\sin\frac{r\pi}{n}[(-1)^r\{B_{20}(\theta_{m\cdot n}(-1)^i+\theta_{0\cdot n}+B_2C_i[\theta_{x\cdot n}]\}-\{B_{20}(\theta_{m.0}(-1)^i$$
$$+\theta_{0\cdot 0})+B_2C_i[\theta_{x\cdot 0}]\}]+D_r(B_{20}-B_2/2)\{S_r[\theta_{m\cdot y}](-1)^i+S_r[\theta_{0\cdot y}]\}$$
$$+\{2K_1(6-D_i)+B_2D_r\}\times\Theta_{i\cdot r}-\frac{12K_1}{\lambda_1}\sin\frac{i\pi}{m}\,\Omega_{ir}=0\,, \tag{46}$$

$$S_i[\mathfrak{M}'_n](-1)^r+S_i[\mathfrak{M}'_0]-\frac{3K_2}{\lambda_2}(4-D_r)\{S_i[\delta_{x\cdot n}](-1)^r-S_i[\delta_{x\cdot 0}]\}$$
$$+\sin\frac{i\pi}{m}[(-1)^i\{B_{10}(\theta'_{m\cdot n}(-1)^r+\theta'_{0\cdot n})+B_1C_r[\theta'_{m\cdot y}]\}-\{B_{10}(\theta'_{0\cdot n}(-1)^r$$
$$+\theta'_{0\cdot 0})+B_1C_r[\theta'_{0\cdot y}]\}]+D_i(B_{10}-B_1/2)\{S_i[\theta'_{x\cdot n}](-1)^r+S_i[\theta'_{x\cdot 0}]\}$$
$$+\{2K_2(6-D_r)+B_1D_i\}\times\Theta'_{i\cdot r}-\frac{12K_2}{\lambda_2}\sin\frac{r\pi}{n}\,\Omega_{ir}=0\,, \tag{47}$$

$$\frac{12K_1}{\lambda_1^2}\sin\frac{i\pi}{m}\{(-1)^iS_r[\delta_{m\cdot y}]-S_r[\delta_{0\cdot y}]\}+\frac{12K_2}{\lambda_2^2}\sin\frac{r\pi}{n}(-1)^rS_i[\delta_{x\cdot n}]$$
$$-S_i[\delta_{x\cdot 0}]\}-\frac{12K_1}{\lambda_1}\sin\frac{i\pi}{m}\Theta_{ir}-\frac{12K_2}{\lambda_2}\sin\frac{r\pi}{n}\Theta'_{ir}$$
$$+\left(\frac{12K_1}{\lambda_1^2}D_i+\frac{12K_2}{\lambda_2^2}D_r\right)\times\Omega_{ir}=-Q_{ir}\,, \tag{48}$$

where

$$\Theta_{ir}=\frac{1}{2}S_r[\theta_{m\cdot y}](-1)^i+\frac{1}{2}S_r[\theta_{0.y}]+C_iS_r[\theta_{x\cdot y}]\,,$$
$$\Theta'_{ir}=\frac{1}{2}S_i[\theta'_{x\cdot n}](-1)^r+\frac{1}{2}S_i[\theta'_{x\cdot 0}]+S_iC_r[\theta'_{x\cdot y}]\,,$$
$$\Omega_{ir}=S_iS_r[\delta_{x\cdot y}]\,,\qquad Q_{ir}=S_iS_r[q_{x\cdot y}]\,,$$
$$D_i=2\left(1-\cos\frac{i\pi}{m}\right),\qquad D_r=2\left(1-\cos\frac{r\pi}{n}\right)$$
$$i,x=0,1,2,\cdots m\,;\; r,y=0,1,2,\cdots n\,.$$

From the above three equations, Θ_{ir}, Θ'_{ir}, Ω_{ir} could be easily solved, and θ_{xy}, θ'_{xy}, δ_{xy} could also be given by the inversion formulas. Thus derived solutions however involve $S_r[\mathfrak{M}_m]$, $S_r[\delta_{my}]$, $C_r[\theta'_{my}]$, $S_r[\theta_{my}]$, θ_{mn}, θ_{m0}, θ'_{mn}, θ'_{m0} and such; which should be determined so as to satisfy the boundary conditions of the edges and the corner points.

5. Case When Four Edges are Hinged

The boundary conditions for the case are represented by (37); and if $B_{10}=B_1/2$, $B_{20}=B_2/2$, then (46), (47) and (48) take the simpler forms as

$$\{2K_1(6-D_i)+B_2D_r\}\times\Theta_{ir}-\frac{12K_1}{\lambda_1}\sin\frac{i\pi}{m}\,\Omega_{ir}=0\,, \tag{49}$$

$$\{2K_2(6-D_r)+B_1D_i\}\times\Theta'_{ir}-\frac{12K_2}{\lambda_2}\sin\frac{r\pi}{n}\,\Omega_{ir}=0\,, \tag{50}$$

$$-\frac{12K_1}{\lambda_1}\sin\frac{i\pi}{m}\,\Theta_{ir}-\frac{12K_2}{\lambda_2}\sin\frac{r\pi}{n}\,\Theta'_{ir}$$
$$+\left(\frac{12K_1}{\lambda_1^2}D_i+\frac{12K_2}{\lambda_2^2}D_r\right)\times\Omega_{ir}=-Q_{ir}\,, \tag{51}$$

from which we have

$$\begin{aligned}\theta_{x\cdot y}&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}\Theta_{ir}\cos\frac{i\pi x}{m}\sin\frac{r\pi y}{n}\\&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}\frac{12K_1}{\lambda_1}Q_{ir}\sin\frac{i\pi}{m}\{2K_2(6-D_r)+B_1D_i\}/A_{ir}\\&\quad\times\cos\frac{i\pi x}{m}\cdot\sin\frac{r\pi y}{n}\,,\end{aligned} \tag{52}$$

$$\begin{aligned}\theta'_{x\cdot y}&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}\Theta'_{ir}\sin\frac{i\pi x}{m}\cos\frac{r\pi y}{n}\\&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}\frac{12K_2}{\lambda_2}Q_{ir}\sin\frac{r\pi}{n}\{2K_1(6-D_i)+B_2D_r\}/A_{ir}\\&\quad\times\sin\frac{i\pi x}{m}\cdot\cos\frac{r\pi y}{n}\,,\end{aligned} \tag{53}$$

$$\begin{aligned}\delta_{x\cdot y}&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}\Omega_{ir}\sin\frac{i\pi x}{m}\cdot\sin\frac{i\pi y}{n}\\&=\frac{4}{mn}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}Q_{ir}\sin\frac{i\pi x}{m}\cdot\sin\frac{r\pi y}{n}\{2K_1(6-D_i)+B_2D_r\}\\&\quad\times\{2K_2(6-D_r)+B_1D_i\}/A_{ir}\,.\end{aligned} \tag{54}$$

where

$$\begin{aligned}A_{ir}&=\frac{12K_1K_2}{\lambda_1^2}\Big[D_i\{2(6-D_r)+\alpha_{21}\}(D_i+\alpha_{12}D_r)\\&\quad+\frac{K_1}{K_2}\beta^2D_r\{2(6-D_i)+\alpha_{12}\}(D_r+\alpha_{21}D_i)\Big]\,,\end{aligned}$$

$$\alpha_{12}=B_2/K_1\,,\qquad \alpha_{21}=B_1/k_2\,,\qquad \beta=\lambda_1/\lambda_2\,,$$

Putting the above results into (13) and (14), we can write the bending moments of the y-th beam in x direction, as follows:

$$\begin{aligned}\left.\begin{matrix}M_{x\cdot x+1}\\M_{x\cdot x-1}\end{matrix}\right\}&=-\frac{24K_1B_2}{mn\lambda_1}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}Q_{ir}D_i\sin\frac{i\pi}{m}\cdot\cos\frac{i\pi x}{m}\cdot\sin\frac{r\pi y}{n}\\&\quad\times\{2K_2(6-D_r)+B_1D_i\}/A_{ir}\pm\frac{48K_1^2}{mn\lambda_1}\sum_{i=1}^{m-1}\sum_{r=1}^{n-1}Q_{ir}D_i\\&\quad\times\sin\frac{i\pi x}{m}\times\sin\frac{r\pi y}{n}\{2K_2(6-D_r)+B_1D_i\}/A_{ir}\,,\end{aligned} \tag{55}$$

6. Numerical Example

The solutions given in the previous article, take the forms of finite double sine or cosine series, so we could carry on the numerical computation without paying any attention to convergency of the series that is required in case of plate problem.

Suppose the loading condition is that a concentrated load of intensity q acts on the center intersection, then

$$Q_{ir} = q \sin \frac{i\pi}{2} \cdot \sin \frac{r\pi}{2}$$

and let

$$\alpha_{12} = \alpha_{21} = 0.5\,, \qquad K_1 = K_2\,, \qquad \beta = 1\,,$$
$$\lambda_1 = \lambda_2 = l/10\,, \qquad m = n = 10\,,$$

Table 1. Values of $M_{x \cdot x+1}$, $M_{x \cdot x-1}$ $(ql/10)$

	$y=1$	$y=2$	$y=3$	$y=4$	$y=5$
$M_{0.1}$	-0.0577	-0.0967	-0.1041	-0.1007	-0.0628
$M_{1.0}$	0.2912	0.5001	0.5368	0.5174	0.3450
$M_{1.2}$	0.1710	0.2737	0.2882	0.2884	0.1574
M_{211}	0.5464	1.0493	1.1736	1.0942	0.9142
$M_{2.3}$	0.4154	0.7622	0.8146	0.7459	0.5856
$M_{3.2}$	0.8294	0.6800	2.0903	2.2255	1.8842
$M_{3.4}$	0.7035	0.3906	1.6065	1.3859	1.3022
$M_{4.3}$	1.0224	0.1488	3.0928	3.9554	3.9057
$M_{4.5}$	0.9471	0.9683	2.7901	3.0014	2.2507
$M_{5.4}$	1.0560	0.2409	0.5247	5.6220	8.2313

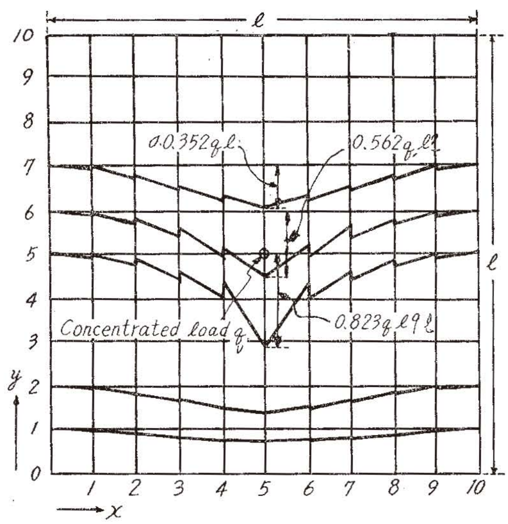

Fig. 3. Variation of bending moments of the beams in x direction with x.

then the maximum deflection which occurs at the center, is easily computed from (54);

$$\delta_{\max} = 0.01396\, ql/K_1 .$$

The bending moments of the beams in x direction are given on Table 1 which tabulates the values about one fourth of the points of intersection because the distribution of the bending moments takes place double symmetrically with respect to both center lines in x and y directions. Fig. 3 also shows how the bending moment goes with the variation of x, the torsional resistance of the beams in y direction makes the diagram into a jagged shape.

7. Conclusion

In analysing the stress problem of the grid plate, the author has developed a method of solving the finite difference equations by means of finite Fourier transforms which are derived from finite integration. Thus obtained solutions are written in finite sine and cosine series, and so there is no problem about convergence of series and infinity of bending moment which may happen when we substitute the equivalent orthotropic plate[4] for the grid work.

The method could also be valid for solving the framed structures which have geometric pattern of members in regular sequence.

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